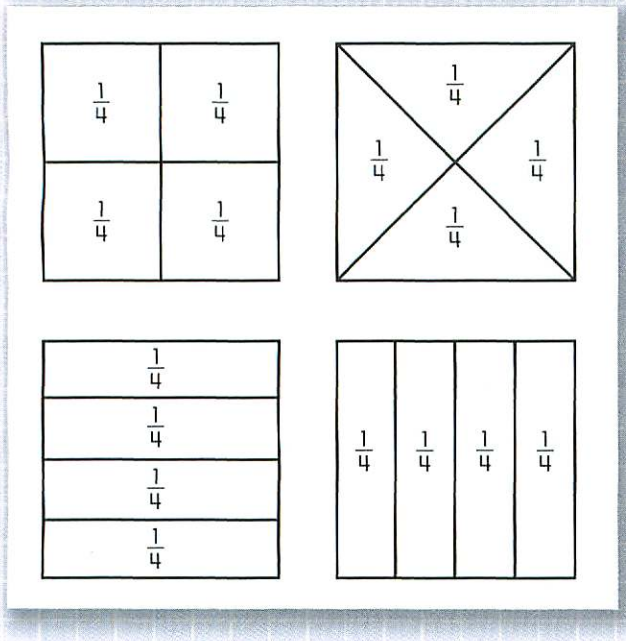
# Fourths: Same or Different?

During Session 2.1, students in this class folded square pieces of paper into fourths and found three different ways to show fourths. Now they consider whether some fourths represent a larger quantity than others or whether all fourths are the same size.

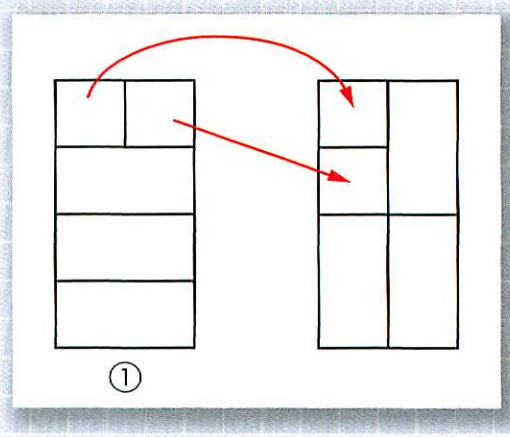



**Teacher:** I have a question for you. Let's say that these pieces of paper are sandwiches and we cut them in different ways. If I gave you a piece from this sandwich [pointing to the first square] or a piece from this other sandwich [pointing to the second square] or a piece from the third sandwich [pointing to the third square], which piece would you choose? Would they be the same?

I don't want you to tell me your answer now. Think about it as you work and come up with a way to explain to all of us how you know.

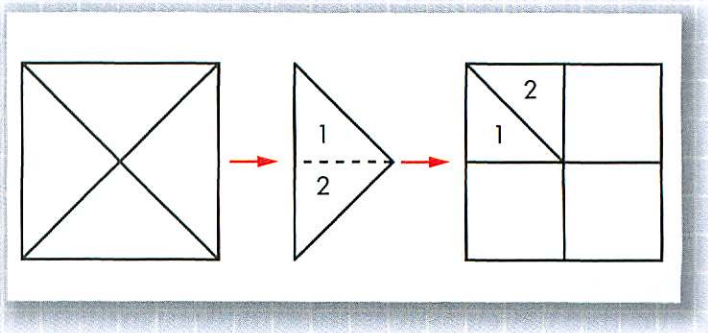
As students start on the problem, the teacher circulates around the room to observe the students at work.

The teacher watches as Paige picks up her scissors and cuts off a rectangular fourth.

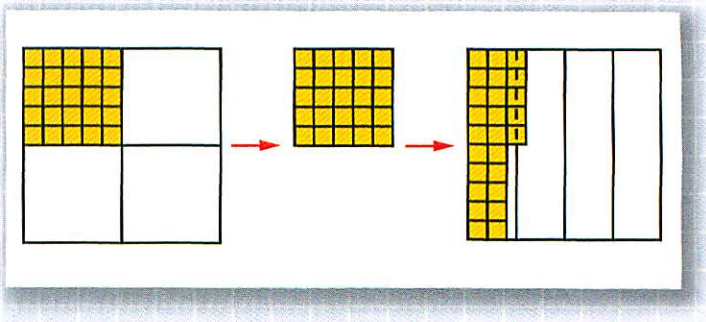



She cuts it in half and places the two parts onto another paper to show a perfect fit over a square fourth.

It takes longer for Paige to see how to do the same with the triangular pieces, but she eventually gets it.

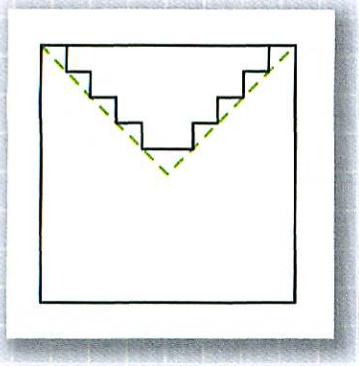



The teacher sees Anita laying out 25 cubes on a fourth in the first square. Then she takes the 25 cubes and lays them out on the second square.



**Teacher:** How many cubes do you need to cover that rectangular piece?

**Anita:** [pointing to the five cubes on the right] I need to cut these five cubes and put the pieces down here so you can see that it's the same, 25.



It's harder for Anita to see how the 25 cubes can fit on the triangular piece, so she cuts squares about the size of a cube face, cuts some in half diagonally, and glues them on.

Anita knows that she has covered the triangle with 25 squares. However, because her cutting isn't as precise as this drawing, there are gaps and overlaps, and because it's hard to see that there are 25 squares (20 whole squares and 10 half squares), her demonstration is less convincing to her classmates.

When the teacher asks Jeffrey what he thinks, he simply places one whole square on top of the other.

**Teacher:** So you're showing that because the squares are the same size, the fourths must be the same size?

**Jeffrey:** [vigorously nodding his head] Yes!

When the class gathers to discuss their findings, these three students share how they are convinced that the pieces are equal. After their presentations, the teacher encourages the class to continue thinking about how these different shapes can be equal.

**Teacher:** But we have three different shapes. How can the pieces be equal?

**Juanita:** Maybe because this (rectangular) piece is the tallest, but it's skinnier than the square and the triangle. The tall pieces are skinnier and the short pieces are fatter.

**Estaban:** It's how you cut it. If you cut the triangle pieces, they fit on the square.

**Teacher:** This is a difficult idea, and you have good ways of thinking about it.

**Jacy:** This proves that it doesn't have to look the same to be the same.

Many second graders do have the resources to take on such a challenging question as whether differently shaped fourths are actually the same quantity. Paige and Anita illustrate two significant strategies to prove that two regions have equal area.

1. Decomposing one region into parts and rearranging the parts over the other region
2. Showing that the same number of units covers both regions

After students have offered their proofs, the teacher asks how different shapes can be equal, challenging the class to think more about their conclusions and, perhaps, voicing confusion that may be shared by some members of the class. This gives Juanita and Esteban further opportunity to verbalize how these shapes are related and gives their classmates the opportunity to think about what they see.

Jacy summarizes the class's findings by observing that "it doesn't have to look the same to be the same," bringing forward the idea that there are different ways to think about what "same" means.